

THE ATTITUDE DETERMINATION OF A THREE AXES STABILIZED SPACECRAFT

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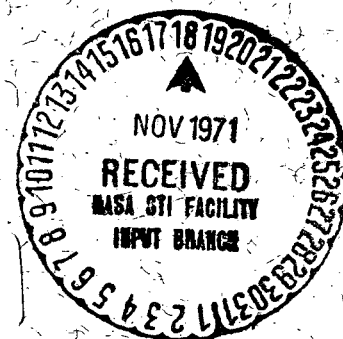
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by

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ABSTRACT

This report presents an approach for attitude determination of a three axis stabilized spacecraft. It describes the employment of two independent vectors whose components with respect to two reference coordinate axes can be known. Such two coordinate axes are right-handed three mutually orthogonal axes systems of which one is fixed in the spacecraft and the other has known orientation in space. The relationship between these two systems are then expressed into mathematical equations with the use of those two reference vectors. By computing the derived equations, the attitude of the spacecraft can therefore be obtained.

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INTRODUCTION

Assume that there is a first coordinate system fixed in the spacecraft and the spacecraft is stabilized about all three axes. Also assume that a second reference coordinate axes is being used whose orientation in space is always known. The attitude control system of the spacecraft is required to provide the stabilization and to maintain the alignment of the first system to the second coordinate axes. Any deviation of the first system from the second one is detected by the sensing instruments on-board the spacecraft. The complete sensor data are then employed to determine two independent vectors in space, such as the sun's position vector and the earth's magnetic field. These vectors are applied to obtain the attitude of the spacecraft. This report presents the analysis of the attitude determination of such a spacecraft and the formulation of required mathematical equations. The application of this approach is also discussed and shown in simple example.

Coordinate Axes Definitions

For convenience, it is necessary to make the following definitions:

(a) The Spacecraft Body Axes.—The spacecraft body coordinate axes x, y, z is a set of right handed mutually orthogonal axes fixed in the spacecraft with their origin at the center of mass. It is referred as first coordinate system mentioned above.

(b) The desired Orientation System.—This system X, Y, Z with its known orientation in space is called the second reference coordinate axes sometimes. The orientation of this orthogonal axes system is always the desired one for the spacecraft body axes. That is, the x, y, z axes are to align with X, Y, Z respectively if there is no attitude error.

(c) **Orbital Coordinate Axes.**—The orbital coordinate axes X_0, Y_0, Z_0 is used only for the spacecraft with circular orbit. This system has its center at the instantaneous location O_0 of the spacecraft with

X_0 = spacecraft's velocity vector

Y_0 = normal to the orbital plane

Z_0 = the downward local vertical defined as the unit vector from the location of the spacecraft to the center of the earth.

(d) The inertial coordinate system is an earth center right handed orthogonal system such that the Z_i axis coincides with the north pole and the $X_i Y_i$ plane is the equatorial plane in which X_i axis passes through the vernal equinox.

The Attitude Determination Equations

It is assumed that the direction cosines of the reference vectors in the desired system X, Y, Z are known and that in the x, y, z axes have been determined. The attitude of the spacecraft can thus be obtained by utilizing the information of these reference vectors.

Suppose that the transformation taking the desired orientation system into the body axes is

$$\begin{aligned}\vec{x} &= A_{11} \vec{X} + A_{12} \vec{Y} + A_{13} \vec{Z} \\ \vec{y} &= A_{21} \vec{X} + A_{22} \vec{Y} + A_{23} \vec{Z} \\ z &= A_{31} \vec{X} + A_{32} \vec{Y} + A_{33} \vec{Z}\end{aligned}\tag{1}$$

Let \vec{v} be a reference vector in space and let

$$[V]_b = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}, \quad \text{and} \quad [V]_d = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}\tag{2}$$

be the components of \vec{v} in the body axes and in the desired coordinate system respectively. Then

$$\begin{aligned}\vec{v} \cdot \vec{x} &= A_{11} \vec{v} \cdot \vec{X} + A_{12} \vec{v} \cdot \vec{Y} + A_{13} \vec{v} \cdot \vec{Z} \\ \vec{v} \cdot \vec{y} &= A_{21} \vec{v} \cdot \vec{X} + A_{22} \vec{v} \cdot \vec{Y} + A_{23} \vec{v} \cdot \vec{Z} \\ \vec{v} \cdot \vec{z} &= A_{31} \vec{v} \cdot \vec{X} + A_{32} \vec{v} \cdot \vec{Y} + A_{33} \vec{v} \cdot \vec{Z}\end{aligned}$$

This means

$$\begin{aligned} v_x &= A_{11} V_1 + A_{12} V_2 + A_{13} V_3 \\ v_y &= A_{21} V_1 + A_{22} V_2 + A_{23} V_3 \\ v_z &= A_{31} V_1 + A_{32} V_2 + A_{33} V_3 \end{aligned} \quad (3)$$

Equation (3) implies that any vector \bar{v} in the X, Y, Z system written in column form, may be transformed into x, y, z system by the application of the matrix $[A]$. That is

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = [A] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad (4)$$

where

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad (5)$$

The matrix $[A]$ is defined by the coefficient A_{ij} ($i, j = 1, 2, 3$) in equation (1) and is now to be determined.

Since $[A]$ is a 3×3 matrix, it requires three reference vectors for obtaining the solution of A_{ij} . Let \bar{D} , \bar{E} , and \bar{F} be such required three vectors. Suppose that $[D]_d$, $[E]_d$ and $[F]_d$ are computed and also that $[D]_b$, $[E]_b$ and $[F]_b$ have been obtained. By applying equation (4) to the vectors \bar{D} , \bar{E} , and \bar{F} respectively, it yields

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = [A] \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = [A] \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (7)$$

and

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = [A] \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \quad (8)$$

Combining these three column matrix equations into a single square matrix equation, gives

$$\begin{bmatrix} D_x & E_x & F_x \\ D_y & E_y & F_y \\ D_z & E_z & F_z \end{bmatrix} = [A] \begin{bmatrix} D_1 & E_1 & F_1 \\ D_2 & E_2 & F_2 \\ D_3 & E_3 & F_3 \end{bmatrix} \quad (9)$$

It may be written in this simple notation

$$[B] = [A] [Q] \quad (10)$$

where

$$[B] = \begin{bmatrix} D_x & E_x & F_x \\ D_y & E_y & F_y \\ D_z & E_z & F_z \end{bmatrix}$$

and

$$[Q] = \begin{bmatrix} D_1 & E_1 & F_1 \\ D_2 & E_2 & F_2 \\ D_3 & E_3 & F_3 \end{bmatrix}$$

If the determinant of $[Q]$ is not zero, then $[Q]$ is non-singular and possesses an inverse $[Q]^{-1}$.

By multiplying the matrix equation (10) with $[Q]^{-1}$, gives

$$[B] [Q]^{-1} = [A] [Q] [Q]^{-1}$$

or

$$[A] = [B] [Q]^{-1} \quad (11)$$

It should be noticed that knowing two reference vectors \vec{D} and \vec{E} the third vector \vec{F} can be defined from them. For simplicity, let \vec{D} and \vec{E} be unit vectors. By defining

$$\vec{F} = \vec{D} \times \vec{E}$$

the column matrix $[F]_d$ and $[F]_b$ are

$$[F]_d = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} D_2 E_3 - D_3 E_2 \\ D_3 E_1 - D_1 E_3 \\ D_1 E_2 - D_2 E_1 \end{bmatrix} \quad (12)$$

$$[F]_b = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} D_y E_z - D_z E_y \\ D_z E_x - D_x E_z \\ D_x E_y - D_y E_x \end{bmatrix} \quad (13)$$

If η is the angle between vectors \vec{D} and \vec{E} , the angle η is calculated from

$$\cos \eta = D_1 E_1 + D_2 E_2 + D_3 E_3 \quad (14)$$

Then the matrix $[Q]$ can be written as

$$[Q] = \begin{bmatrix} D_1 & E_1 & D_2 E_3 - D_3 E_2 \\ D_2 & E_2 & D_3 E_1 - D_1 E_3 \\ D_3 & E_3 & D_1 E_2 - D_2 E_1 \end{bmatrix} \quad (15)$$

and its determinant is

$$\det [Q] = |Q| = \sin^2 \eta$$

If $\eta \neq 0$, then

$$|Q| \neq 0$$

The matrix $[Q]$ is non-singular and has an inverse $[Q]^{-1}$ where

$$[Q]^{-1} = \frac{1}{\sin^2 \eta} \begin{bmatrix} D_1 - E_1 \cos \eta & D_2 - E_2 \cos \eta & D_3 - E_3 \cos \eta \\ E_1 - D_1 \cos \eta & E_2 - D_2 \cos \eta & E_3 - D_3 \cos \eta \\ D_2 E_3 - D_3 E_2 & D_3 E_1 - D_1 E_3 & D_1 E_2 - D_2 E_1 \end{bmatrix} \quad (16)$$

Now equation (11) becomes

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \frac{1}{\sin^2 \eta} \begin{bmatrix} D_x & E_x & D_y E_z - D_z E_y \\ D_y & E_y & D_z E_x - D_x E_z \\ D_z & E_z & D_x E_y - D_y E_x \end{bmatrix} \begin{bmatrix} D_1 - E_1 \cos \eta & D_2 - E_2 \cos \eta & D_3 - E_3 \cos \eta \\ E_1 - D_1 \cos \eta & E_2 - D_2 \cos \eta & E_3 - D_3 \cos \eta \\ D_2 E_3 - D_3 E_2 & D_3 E_1 - D_1 E_3 & D_1 E_2 - D_2 E_1 \end{bmatrix} \quad (17)$$

This directly shows

$$A_{11} = \{(D_x D_1 + E_x E_1) - (D_x E_1 + D_1 E_x) \cos \eta + (D_y E_z - D_z E_y) (D_2 E_3 - D_3 E_2)\} / \sin^2 \eta$$

$$A_{12} = \{(D_x D_2 + E_x E_2) - (D_x E_2 + D_2 E_x) \cos \eta + (D_y E_z - D_z E_y) (D_3 E_1 - D_1 E_3)\} / \sin^2 \eta$$

$$A_{13} = \{(D_x D_3 + E_x E_3) - (D_x E_3 + D_3 E_x) \cos \eta + (D_y E_z - D_z E_y) (D_1 E_2 - D_2 E_1)\} / \sin^2 \eta$$

$$\begin{aligned}
A_{21} &= \{ (D_y D_1 + E_y E_1) - (D_y E_1 + D_1 E_y) \cos \eta + (D_x E_x - D_x E_z) (D_2 E_3 - D_3 E_2) \} / \sin^2 \eta \\
A_{22} &= \{ (D_y D_2 + E_y E_2) - (D_y E_2 + D_2 E_y) \cos \eta + (D_x E_x - D_x E_z) (D_3 E_1 - D_1 E_3) \} / \sin^2 \eta \\
A_{23} &= \{ (D_y D_3 + E_y E_3) - (D_y E_3 + D_3 E_y) \cos \eta + (D_x E_x - D_x E_z) (D_1 E_2 - D_2 E_1) \} / \sin^2 \eta \\
A_{31} &= \{ (D_z D_1 + E_z E_1) - (D_z E_1 + D_1 E_z) \cos \eta + (D_x E_y - D_y E_x) (D_2 E_3 - D_3 E_2) \} / \sin^2 \eta \\
A_{32} &= \{ (D_z D_2 + E_z E_2) - (D_z E_2 + D_2 E_z) \cos \eta + (D_x E_y - D_y E_x) (D_3 E_1 - D_1 E_3) \} / \sin^2 \eta \\
A_{33} &= \{ (D_z D_3 + E_z E_3) - (D_z E_3 + D_3 E_z) \cos \eta + (D_x E_y - D_y E_x) (D_1 E_2 - D_2 E_1) \} / \sin^2 \eta
\end{aligned} \tag{18}$$

From the above equations the deviation of the body axes from the desired orientation system can therefore be known. For FORTRAN program, equation (17) is recommended.

Euler Angle Transformation

The attitude error of the body axes x, y, z from the desired orientation system X, Y, Z can be represented by a sequence of three Euler angle rotations. The first is that a rotation about axis p through angle ψ , the second about axis q through angle ϕ , and the third about axis r through angle θ . Thus there are twelve different sequence of Euler angle rotations that can be formed such as 121, 123, 131, etc. However, any one of them is available for choosing to represent the spacecraft attitude. For instance, let the sequence 312 be selected. That is, yaw angle ψ around the Z axis, roll angle ϕ about the displaced X axis, and pitch angle θ about the doubly displaced Y axis (see figure 1). The notation to be used for the matrix which performs each of these three Euler rotation is

$$R_z(\psi) = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

Therefore

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = R_y(\theta) R_x(\phi) R_z(\psi) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad (19)$$

By comparing equation (19) with equation (4) it gives

$$[A] = R_y(\theta) R_x(\phi) R_z(\psi)$$

or

$$\begin{aligned} [A] &= \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos \theta \cos \psi - \sin \theta \sin \phi \sin \psi & \cos \theta \sin \psi + \sin \theta \sin \phi \cos \psi & -\sin \theta \cos \phi \\ -\cos \phi \sin \psi & \cos \phi \cos \psi & \sin \phi \\ \sin \theta \cos \psi - \cos \theta \sin \phi \sin \psi & \sin \theta \sin \psi - \cos \theta \sin \phi \cos \psi & \cos \theta \cos \phi \end{bmatrix} \quad (20) \end{aligned}$$

The roll angle ϕ , pitch angle θ , and yaw angle ψ can thus be derived

$$A_{13} = -\sin \theta \cos \phi$$

$$A_{23} = \sin \phi$$

$$A_{33} = \cos \theta \cos \phi$$

and

$$A_{21} = -\sin \psi \cos \phi$$

Hence

$$\phi = \sin^{-1} (A_{23}) \quad (21)$$

$$\theta = \tan^{-1} \left(-\frac{A_{13}}{A_{33}} \right) \quad (22)$$

of the angle of ϕ is limited to $-90 \leq \phi \leq 90$ then $\cos \phi$ is always positive and

$$\sin \psi = -\frac{A_{21}}{\cos \phi} = -\frac{A_{21}}{\sqrt{1 - A_{23}^2}}$$

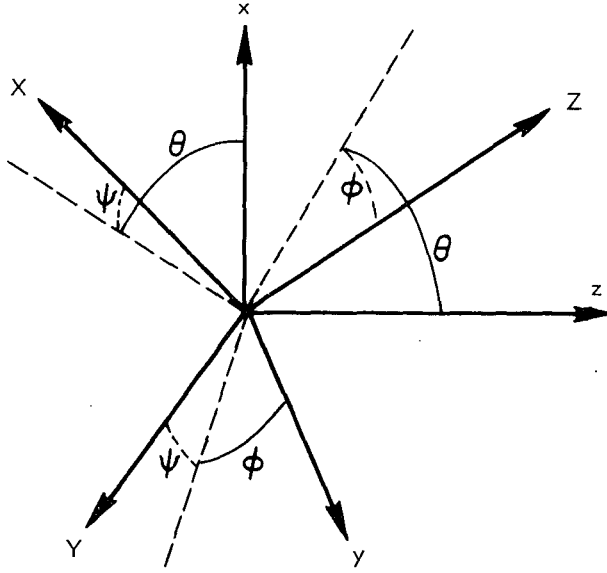


Figure 1. Attitude Error Representation: Roll ϕ , Pitch θ , Yaw ψ

Thus

$$\psi = \sin^{-1} \left(-\frac{A_{21}}{\sqrt{1 - A_{23}^2}} \right) \quad (23)$$

Application

There are three reference vectors whose components in the desired orientation system can always be known. They are the sun position vector, the earth magnetic field, and the downward local vertical. In order that Equation (17) or (18) can be applied, the direction cosines of each reference vector in the body axes must also be found. At present, three kinds of sensors have been developed for this purpose. The sun sensor measuring the direction cosines of the sun position vector, the earth sensor providing the information of the downward local vertical, and the magnetometer giving the components of earth magnetic field. Since most spacecraft contains only two kinds of sensor because of limited space, the given information can thus provide two available reference vectors to be used instead of three.

It should be noticed that the desired orientation system is different from one spacecraft to the other. The transformation for calculating the components of the reference vector in that system must be formulated according to the requirement of each spacecraft.

For example if a spacecraft is in a circular orbit with an on-board magnetometer, the components of the earth magnetic field \vec{M} in the body axes will be measured. Let M_x , M_y , and M_z be such three components. Then

$$\vec{D} = \vec{M}/|M|$$

and

$$[D]_b = \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \frac{1}{|M|} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \quad (24)$$

The spacecraft position unit vector \vec{R} is known from orbit determination and will be indicated in orbit 3A tape. If R_1 , R_2 , and R_3 are the three direction cosines of vector \vec{R} in the inertial coordinate system at the time when the magnetometer makes its measurements, then

$$\begin{aligned} \cos \sigma &= R_1 / \sqrt{R_1^2 + R_2^2} \\ \sin \sigma &= R_2 / \sqrt{R_1^2 + R_2^2} \\ \cos \nu &= R_3 \\ \sin \nu &= \sqrt{R_1^2 + R_2^2} \end{aligned} \quad (25)$$

Since the instantaneous position O_0 of the spacecraft is known, a local coordinate system X_e , Y_e , Z_e with its origin at O_0 can be defined where

\vec{X}_e is pointing toward north

\vec{Y}_e is pointing toward east

$\vec{Z}_e = \vec{Z}_0$

At the point O_0 the direction cosines (M_N , M_E , M_V) of the magnetic field \vec{M} in the X_e , Y_e , Z_e axes can be derived from the actual knowledge of the field. Suppose now the inertial coordinate system is the desired orientation for the spacecraft body axes. The (M_1 , M_2 , M_3) of \vec{M} in that system should be calculated by using the following coordinate transformation

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -\cos \nu \cos \sigma & -\sin \sigma & -\sin \nu \cos \sigma \\ -\cos \nu \sin \sigma & \cos \sigma & -\sin \nu \sin \sigma \\ \sin \nu & 0 & -\cos \nu \end{bmatrix} \begin{bmatrix} M_N \\ M_E \\ M_V \end{bmatrix} \quad (26)$$

Hence

$$[D]_d = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} \quad (27)$$

Also assume that the spacecraft has on-board earth sensor providing the information for computing the components L_x , L_y , and L_z of the downward local vertical \vec{Z}_0 . Then

$$\vec{E} = \vec{Z}_0 = -\vec{R}$$

$$[E]_b = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix}$$

and

$$[E]_d = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = - \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

Knowing $[D]_b$, $[D]_d$, $[E]_b$, and $[E]_d$, equation (18) can be computed immediately.

Next suppose that desired orientation of the spacecraft is changed from the inertial coordinate system to the orbital coordinate axes because of the purpose for special experiment. The column matrices $[D]_b$, $[E]_b$ will remain in the same form, but $[D]_d$ and $[E]_d$ are different. In this case

$$[D]_d = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \sin i & \cos i & 0 \\ -\cos i & \sin i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_N \\ M_E \\ M_V \end{bmatrix}$$

where i is the orbit inclination (i.e. angle between orbit plane and equatorial plane), and

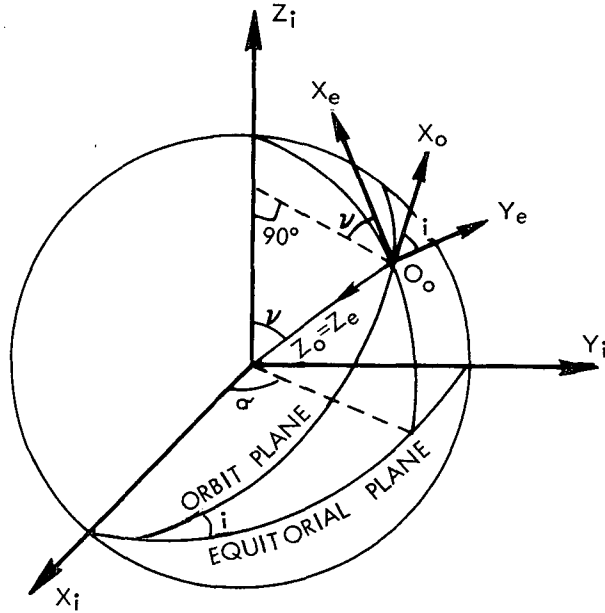


Figure 2. The X_e , Y_e and Z_e Axes in the Local Coordinate System

$$[E]_d = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Substituting these values into equation (17) or (18) the problem of attitude determination is then solved.

CONCLUSIONS

The approach described here may well be suited for attitude determination of non-spinning spacecraft consisting of any two of the three on-board instruments; namely, the sun sensor, the magnetometer, and the earth sensor for providing the informations of two reference vectors. The only restriction for this approach is that the two selected reference vectors must not be parallel.

The idea of this approach was first practiced on Delta-PAC in Jan. 1969 when the author was assigned to formulate the attitude determination equations and error prediction formulae for Delta-PAC project. The derived equation were exercised with actual flight data obtained from Delta-PAC

after it was launched on Aug. 9, 1969. The results determined the attitude behavior of Delta-PAC and evaluated its control system. However, the Delta-PAC attitude determination equations were limited to be used only for that particular spacecraft and cannot be directly applied to others if not modified. Therefore, the more generalized attitude determination equations for a three axes stabilized spacecraft seem to be needed and are contained in this report which are expected to be applicable for most non-spinning spacecraft.

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